Circular motion, Center of Gravity, and Rotational Mechanics

Rotation and Revolution

- Every object moving in a circle turns around an axis.
- If the axis is internal to the object (inside) then it is called a rotation.
  - Ex. Earth spinning, merry-go-round
- If the axis of rotation is external to the object (outside) then the object is revolving.
  - Ex. Earth revolving around the sun, duck-duck-goose

Period, frequency

- The time it takes for one full rotation or revolution is the period.
  - Earth's period of rotation is 24 hours
  - Earth's period of revolution is 365.25 days
- The frequency is the number of revolutions or rotations in a period of time.
  - Idling car at 1800rpm = 1800 revolutions per minute (30 per second!)
- $T = 1/f$ where T is the period and f is the frequency.
- Periods are measured in seconds and frequency in hertz (Hz)

Rotational Speed

- Rotational speed is the speed at which it is moving in a circle, also can be called angular speed.
- All parts on a spinning object have the same rotational speed.
- Usually measured in rotations per unit of time like rpm (revolutions per minute)
- Can also be thought of as degrees around a circle per unit of time.

Linear Speed

- Linear speed is the instantaneous speed that an object that is rotating or revolving has.
- This can also be called tangential speed since the direction of motion is always tangent to the circle.
- It is a straight line speed and given by the equation $v = 2\pi r/T$. ($2\pi r$ is the circumference of the circle and T is the period to go around)
- Farther away from the center, the greater the radius!
- One object is at the edge of a merry-go-round while another is half-way out from the center. Which has the greater tangential speed? Angular speed?

Centripetal Acceleration

- Objects can move in a circle with a constant speed, but they are still accelerating.
- This acceleration is always directed toward the center of the circle.
- This acceleration is the centripetal acceleration.
- $a_c = v^2/r$ where v is the linear speed and r is the radius.
- Units are m/s^2

Centripetal Force

- Since there is a centripetal acceleration, there must be a force on the object.
- This force must be pushing/pulling toward the center.
- This is the centripetal force.
- $F_c = ma_c$ or $F_c = mv^2/r$, the units are N
- This force is overcoming the inertia of the object, since the inertia wants it to continue to travel in which direction?
- A straight line.

Centrifugal Force

- The centrifugal force is a “perceived” force. It is not real.
- Since the inertia of the object wants to keep the object going in a straight line, it feels like the object has a force pushing it away from the center.
- THERE IS NO FORCE!!! It just feels like there is!!
- This is why you would feel like you are pressed to the floor in a rotating space station.
- Its really the space station pushing on you with centripetal force to move you in a circle.

Center of Gravity

- The center of gravity is the point where the objects average weight is located.
- This does not need to be located within the object. For instance a doughnut, or a table.
- This need to be calculated or discovered experimentally.
- Also referred to as the center of mass. For most objects it is usually the same spot, unless the object has an uneven gravity distribution.
  - aka, the moon, mercury, skyscrapers

Determining Center of Gravity

- For a symmetrical object is easy to find. For a geometric shape, there is a simple formula.
- For an irregular object you need to hang it from different points with a weight to see where straight down is.
- When you do this several times you can see where the lines cross and that will be the center of gravity.

Motion

- The path that a projectile will take is actually the path that the center of mass of the object will take.
- An irregularly shaped object may tumble, but its center of mass will follow the calculated path.
- We deal with forces and paths with how they effect the center of mass. It is just as good as if we were to take into account the total size.
- For instance the Earth and the Sun interact the same way whether we look at all of their mass concentrated at a central point or not.
- It makes life much easier!

Toppling and Stability

- An object will topple if its center of mass moves out beyond its base.
- As long as its center of mass is over its base it can be balanced.
- Even if an object is balanced, it may not be stable.
- An object is stable if its center of mass would need to be raised by any movement.
- It is unstable if any movement would cause the center of mass to fall.

Torque

- Torque is not just a force, it is a force with leverage.
- The leverage comes from the distance that the force is being applied from the center of rotation.
- The distance from the center of rotation is called the lever arm.
- Only the component of the force perpendicular to the lever arm will produce a torque.
- Objects can be balanced by balancing the torques on the opposite sides of the center of rotation.
- Again, objects topple if their center of gravity is beyond their base because that creates a torque.

Torque Calculations

- $\tau = F_{\perp} \times d$
- Where τ = torque
- $F_{\perp}$ = the perpendicular component of the force.
- d = the distance from the center of rotation that the force is being applied.
- Units for Torque are Nm (Newton meters)

Rotational Inertia

- Rotational Inertia is an objects resistance to changes in rotational motion.
- Torques are required to change the rotational motion of objects.
- Rotational inertia is not just dependent upon mass, but the location of the mass is important.
- The greater the distance the distribution of the mass of the object is from the center, the greater the rotational inertia.
- Ex. Kids on a merry-go-round, choking up on a bat

Formulas for Rotational Inertia

- Each object has a different rotational inertia depending on its shape and axis of rotation.
- We will not be calculating them here, but some equations would be provided if needed.
- Ex. solid sphere rotating about its CG, $I=2/5mr^2$
- A simple pendulum, $I = mr^2$
- A rolling hoop, $I=mr^2$
- A spinning hoop, $I=1/2mr^2$
- Units are $kg\ m^2$

Rolling

- The object that has its mass concentrated toward the center will accelerate down the incline the fastest.
- Which will roll faster down an incline, a solid cylinder or a hollow one?
- Does the mass or radius matter?
- NEVER, because the hollow cylinder always has the greater rotational inertia.
- What about two solid cylinders that have different mass or radius?

Angular Momentum

- Just like objects moving in a straight line have momentum, objects moving in a circle have momentum until an outside force changes it.
- This momentum is called angular momentum.
- For small objects at a large radius, the angular momentum is equal to $L = mvr$
- Where L = the angular momentum
- m is the mass, v the linear velocity, and r is the radius of rotation.
- Units are kg m^2/s

Conservation of AM

- Angular momentum is conserved!!!
- Sooooooooooooooooooooooo…………
- If your radius decreases, what happens to your velocity?
- For larger masses, your angular momentum is the moment of inertia times the angular speed.
- The greater the moment of inertia, the smaller the rotational speed.
- Figure skaters, dancers, gymnasts, divers, cats etc. know this!!!